Fluid flow, Newton's second law and river rescue

Michael J O'Shea

Cardwell Hall, Department of Physics, Kansas State University, Manhattan, KS 66506-2601, USA

E-mail: mjoshea@phys.ksu.edu

Abstract
We consider the situation of a boat pinned or wrapped against a rock by moving water in a river. The force exerted by moving water is calculated and the force required to extricate the boat is estimated. Rafts, canoes and kayaks are each considered. A rope system commonly employed by river runners to extricate a boat is analysed. This system includes a mechanical advantage z-drag and a self-equalizing tie-off, and the tensions in various parts of the system are calculated. Introductory undergraduate physics is used throughout this work.

M This article features online multimedia enhancements

Introduction

During the last two decades the number of people running whitewater rivers has grown dramatically. With this growth has come an appreciation of the force that flowing water can exert on an object and the establishment of clinics to teach river safety and rescue. Educational organizations [1–3] provide training courses for professional rescuers employed by local fire/rescue agencies, and for private river runners.

Whitewater canoes and kayaks contain significant floatation in the form of airbags and will not sink if they fill with water except under exceptional circumstances such as a pin against an undercut rock. They are usually constructed so that they have a smooth semi-rigid outer layer of plastic material. Rafts are constructed of a rubber-coated nylon fabric. Inflated tubes form the outside of the raft and the floor is flexible and usually inflatable. The raft can be manoeuvered by a team of people with paddles or rowed by a single person with oars. Examples of boats are shown in figure 1 and in an ancillary file available from the electronic version of the journal.

Whitewater kayaking is popular in Europe and the United States as evidenced by the large number of guidebooks covering these areas [4]. While canoeing (Canadian style) is popular throughout the world, whitewater canoeing has only gained in popularity in the United States and Canada. Whitewater rafting first became popular in the United States and has now spread to many parts of the world. Commercial companies offer guided raft trips on whitewater rivers in the United Kingdom, France, Switzerland, Austria, Canada, United States, Middle and South America and many other places [5, 6]. A formal rafting curriculum has been developed by the Tyrolean Rafting Association (Austria) [6].

Occasionally a raft may hit a rock and be pinned or wrapped [6, 7] against it by moving water, see figure 1. Similar problems can occur with canoes and kayaks. We look at the force exerted by moving water, determine the force required to extricate a pinned or wrapped boat

Figure 1. Top left: a Canadian style open canoe on the Colorado River, Arizona. Top right: a paddle raft on the Green River, Utah. Bottom left: kayakers on the Main Salmon River, Idaho (courtesy T J Hittle). Bottom right: a wrapped oar raft on the Jarbridge/Bruneau River, Idaho, USA (courtesy A Laakmann).

and calculate the tension in various parts of a rope system used for river rescue. This article should not be taken as instruction on how to use such a system in a real river rescue situation. There are dangers and judgment calls inherent in a river rescue situation that can only be learned by taking a hands-on class in river rescue.

Force exerted by water on a boat

Figures 2(a) and (b) show a raft being pinned against a rock by moving water. A more serious situation occurs when the raft is turned on its side against the rock as shown in the last photograph of figure 1 and in figure 2(c). In such a situation the raft is said to be 'wrapped' since occasionally the ends of the raft are actually wrapped around the sides of the rock. Not all pinned rafts will wrap. If the pinned raft is to wrap, water must flow over the upstream tube. There are several reasons why this might occur:

- Water piles against the upstream tube of the raft and flows over the top of it as indicated in figure 2(d).
- The raft has a smooth curved cross-section that may allow some streamline flow of the deeper water under it as shown in figure 2(d). In order for the water to follow the curved cross-section of the raft, there must be a force exerted on the water by the raft that is directed towards the centre of curvature of the streamlines *that are in contact* with the raft [8, 9]. By Newton's third law, the force exerted on the raft by the water, $\boldsymbol{F}_{\mathrm{SL}}$, is in the opposite direction. Thus $\boldsymbol{F}_{\mathrm{SL}}$ pulls downward on the upstream tube, and this force is shown in figure 2(d).

Once water flows over the upstream tube, the upstream side of the raft is forcefully pushed downward and the raft tilts. People and any loose gear tend to slide and tumble to the upstream side of the raft, causing the raft to tilt further. The upstream tube of the raft becomes submerged and the raft tips sideways (rotates by an angle $\approx 90^\circ$ about its long axis, see figure 2(c)). The underside of the raft is now pushed against the rock and the raft is wrapped. The submerged part of the raft presents an area A to the flowing water. In a time Δt a volume V of water moving with velocity v, shown in figure 2(e), is brought to a stop as it collides with the raft. This volume has a cross-sectional area A and extends a distance

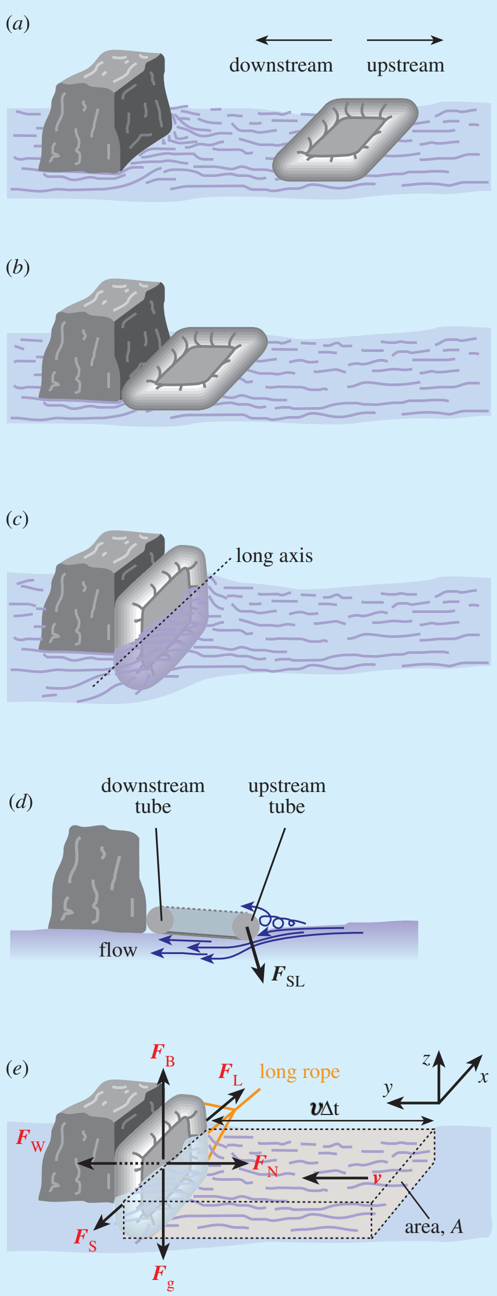

Figure 2. (*a*) A raft floats down a river, (*b*) strikes a rock and pins sideways against the rock, and water rushes over the upstream tube, (*c*) the raft tips sideways (rotates about its long axis) and wraps on the rock. (*d*) An enlarged cross-section of the raft as it strikes the rock and *before it wraps*. Surface water piling against the side of the raft and deeper water flowing in a streamline fashion under the raft are shown along with a possible force $\boldsymbol{F}_{SL}$. (*e*) Free body diagram of the raft *after it wraps*. The upstream face of the rock is in the x–z plane.

$d = v\Delta t$ upstream of the raft. The y-component of the change in momentum of this water, ΔP_y, is $(0 - v)m$ where $m = \rho V = \rho v \Delta t A$ and ρ is the density of water. Newton's second law gives the force exerted *on the water* by the wrapped raft,

$$F'_W = \frac{\Delta P_y}{\Delta t} = -\rho v^2 A, \qquad (1)$$

and is along the $-y$ direction. The force of interest here is the force F_W exerted *on the raft* by the water. By Newton's third law this is $F_W = -F'_W$ [1]. This force is not directly related to the force F_{SL} discussed earlier. F_W results from an abrupt change in momentum of moving water while F_{SL} results from a fluid that is flowing in a streamline fashion [9].

Equation (1) also applies to other whitewater boats such as canoes and kayaks. Table 1 gives the estimated area presented by a boat to the flowing water, assuming the boat is half submerged. These areas are estimated for a 16 ft (4.9 m) NRS E-160 self-bailing raft from 'Northwest River Supplies', a whitewater Viper 12 canoe from 'Mohawk Canoes', and a Dancer whitewater kayak from 'Perception Kayaks'. The force F_W is given for each boat assuming $v = 1.5\ \mathrm{m\,s^{-1}}$, a reasonable value for a rapid that is not steep.

Figure 2(*e*) shows a free body diagram for the raft. A buoyant force $\boldsymbol{F}_B$, a weight force $\boldsymbol{F}_g$, a force $\boldsymbol{F}_W$ exerted by moving water, and a normal force $\boldsymbol{F}_N$ exerted by the rock on the raft are present. If a rope is used to exert a force $\boldsymbol{F}_L$ on the raft along x to pull it off the rock, a static friction force $\boldsymbol{F}_S$ will also be present and will oppose it. To pull the boat off the rock, the force $\boldsymbol{F}_L$ must be greater than the maximum value of the static friction, $F_{S,M} = \mu F_N$. If the upstream face of the rock is vertical then $\boldsymbol{F}_N$ points upstream, and Newton's second law in the form $\boldsymbol{F}_N - \boldsymbol{F}_W = 0$ gives $F_{S,M} = \mu F_W$.

μ is estimated in a separate experiment in which a piece of rubber similar to raft material, or a piece of hard plastic material similar to canoe or kayak construction material, is held down by a mass m on a horizontal, flat, granite rock. The maximum force F_{max} that can be applied without

[1] The form of F_W may also be obtained if one has the physical insight to deduce that the important physical quantities determining F_W are ρ, v and A. F_W is then written as $\rho^\alpha v^\beta A^\gamma$ and the constants α, β and γ can be determined from dimensional arguments.

Table 1. Calculated values of F_W and $F_{S,M}$ and the estimated value of μ for a wrapped boat. A water velocity of 1.5 m s^{-1} is assumed. The area A presented to the flowing water is given assuming the boat is half submerged. The tensions at several points in the z-drag are also given assuming $F_L \approx F_{S,M}$. The error bar on μ, $F_{S,M}$, F_G and F_{Anch} is ±5%.

Boat	Area (m^2)	F_W (kN)	μ	$F_{S,M}$ (kN)	F_G (kN)	F_{Anch} (kN)	n
Raft	4.4	9.90	0.72	7.13	2.38	4.75	6
Canoe	0.95	2.14	0.57	1.22	0.41	0.81	1
Kayak	0.65	1.46	0.57	0.83	0.28	0.56	1

moving the rubber or plastic is then determined and $\mu = F_{max}/mg$. Calculated values of μ and $F_{S,M}$ are given in table 1. The values of F_W and consequently $F_{S,M}$ are large for the raft and much smaller for the canoe and kayak due primarily to their smaller area.

Note that (i) the values of μ depend on the nature of the two surfaces in contact and, in particular, on their roughness, (ii) it is assumed that only friction stops the raft being pulled off, i.e. the occasional possibility of a raft part being caught on an outcropping of rock is ignored, and (iii) the required value of F_L ($\approx \mu F_W$) is strongly dependent on the water velocity. If the velocity is doubled, F_W, $F_{S,M}$ and the required value of F_L are increased by a factor of $2^2 = 4$. Points (i) and (iii) together indicate that the required F_L can vary significantly depending on the situation. If good estimates of μ and v are made, a reasonable estimate for F_L can be made for a particular situation.

The mechanical advantage 'z-drag' and the self-equalizing tie-off

Most whitewater river runners carry enough basic components to create a mechanical advantage system called a 'z-drag' [7] and a self-equalizing tie-off to a boat. Figure 3 gives an overall view of such a rope system attached to a wrapped raft.

Several knots are used in the rope system and we refer the interested reader to reference [10] for an introduction to knots. A rope is used to create the tie-off, span the distance from the raft to the riverbank (the long rope) and create the z-drag portion of the system. Prusik loops are used and are created by tying the ends of a prusik cord together using a 'double fisherman' knot. These loops are attached to the rope using a 'prusik' knot. This knot slides on the rope when it is loose, allowing adjustment of the rope system, and grips tightly when it is under tension.

We discuss several important aspects of the rope system of figure 3 in turn.

The mechanical advantage of the z-drag

The z-drag is named after the letter 'z' formed by the section of rope ABCG. This section of rope is a continuous piece that slides freely through the pulley wheels at B and C. A prusik loop is used to attach the karabiner on the pulley wheel at C to the long rope at point A. A group of people on the river bank at G pull on the rope with a force $\boldsymbol{F}_G$ creating tensions $\boldsymbol{F}_L$ in the long rope and $\boldsymbol{F}_{PR}$ in the prusik cord. We assume the pull $\boldsymbol{F}_G$ is roughly in-line with the long rope so that the angles between the various sections of rope in the z-drag are small and can be set equal to zero. Applying Newton's second law firstly to the free-body diagram of the knot at A gives $\sum F_x = F_G + 2F_{PR} - F_L = 0$, and secondly to the free-body diagram for the pulley–karabiner combination at C gives $\sum F_x = 2F_G - 2F_{PR} = 0$. Eliminating F_{PR} between these two equations gives[2]

$$F_L = 3F_G. \tag{2}$$

Thus the 'z-drag' is a 3:1 mechanical advantage system. Taking F_L to be 7.13 kN, the value when static friction holding the raft in place on the rock is about to be overcome, equation (2) gives $F_G = 2.38$ kN.

The self-equalizing tie-off

The strongest connection points on the raft are the metal D-rings, so named because of their shape. A self-equalizing tie-off is used to attach to as many

[2] This result may be deduced directly by choosing the system to be the knot at A + karabiner and pulley wheel at C + the prusik loop connecting them.

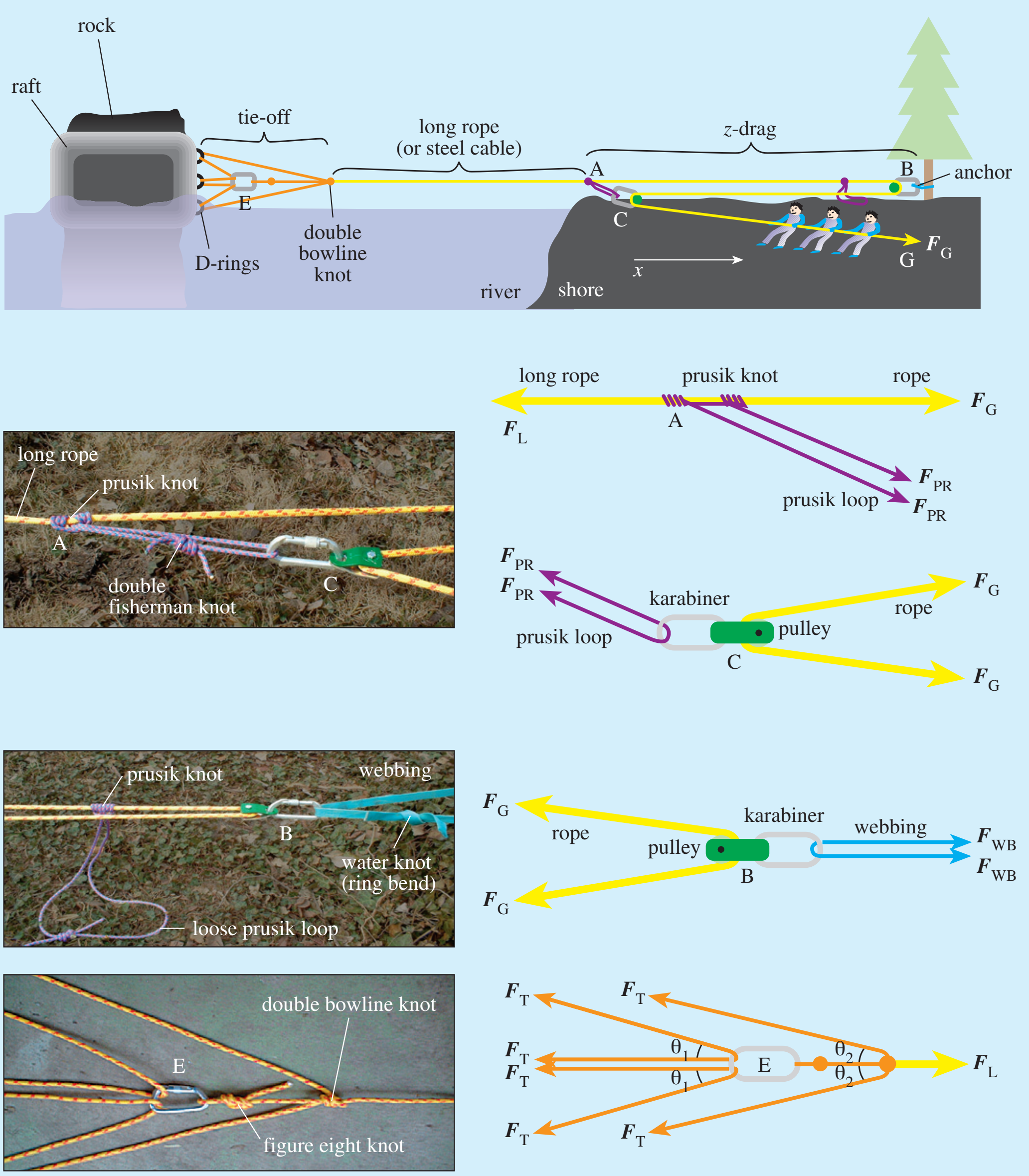

Figure 3. The z-drag mechanical advantage system attached to the raft via a 'long' rope with a self-equalizing tie-off. The rope is orange, prusiks are purple, and the webbing is turquoise to distinguish them. Photographs on the left show various points in the rope system, and the free body diagrams on the right show the forces at various points. At A the system on which the forces are indicated is the prusik knot. At B the system is the karabiner and pulley together (similarly for C). At E the system is the karabiner and the rope extending from it to the double bowline knot. The details of the tie-off are not drawn to scale.

D-rings as possible. The rope length is chosen so that θ_1 and θ_2 are both well below 45°. To determine the tension F_T in the tie-off rope, the karabiner at E and the piece of rope extending from it to the double bowline knot are taken to be the system, and a free-body diagram is shown in figure 3. Assuming the distance from the double bowline to karabiner E is small compared with the length of the tie-off (from the double bowline to the raft), then $\theta_1 \approx \theta_2 \approx \theta$. Newton's second law gives $\sum F_x = F_L - 4F_T\cos\theta - 2F_T = 0$ and

$$F_T = \frac{F_L}{2(1+2\cos\theta)}. \quad (3)$$

Taking F_L to be 7.13 kN and choosing $\theta \simeq 25°$ gives $F_T = 1.27$ kN. The force on each D-ring, $F_{DR} = 2F_T = 2.54$ kN and is significantly less than the total force of 7.13 kN applied to the raft. Note that a self-equalizing tie-off can also be set up with a webbing sling instead of a rope. See the ancillary file available from the electronic version of the journal for detail on webbing tie-offs.

Tension in the anchor webbing and the prusik cord

The webbing connecting karabiner B to the tree has a large surface area compared with a rope and reduces the risk of local damage to the tree. Applying Newton's second law to the free-body diagram for the pulley–karabiner combination at B gives $F_{WB} = F_G$ and for the case of the raft F_{WB} is 2.38 kN. The force on the anchor tree, $F_{Anch} = 2F_{WB} = 4.75$ kN since there are two strands of webbing going to the tree. The webbing should be as low as possible on the tree to minimize torque about the base of the tree.

The tension in the prusik loop connecting the karabiner at C to the long rope at A can be deduced from the above results and is $F_{PR} = F_G = 2.38$ kN. A second loose prusik loop (shown in figure 3) may be used to lock off the long rope under tension by connecting this prusik to the karabiner at B. The people pulling may now rest and the z-drag can be adjusted. The tension in this prusik cord will be $F_L/2 = 3.56$ kN. The above calculation can be repeated for the canoe and kayak, resulting in smaller tensions than for the raft due primarily to the lower surface area of these boats, see table 1.

Estimating the number of people required to pull on the rope

To estimate the number of people required to pull on the rope at G, the force, F_{G1}, with which one person of mass m_p pulls on the rope must first be determined. Figure 4 shows a person pulling at G and a free body diagram for this person. Since the person pulls on the rope to the right with a force F_{G1}, the rope pulls to the left on the person with the same force and this force is shown in the free body

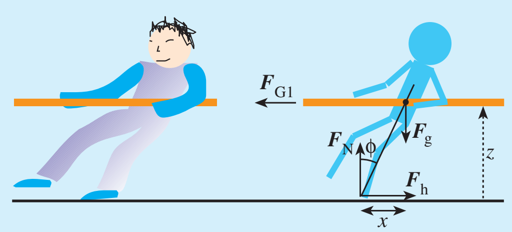

Figure 4. A person on the riverbank at G pulling on a rope (left) and a free-body diagram (right) isolating the forces on the person. The black dot (•) marks the centre of mass of the person.

diagram. Other forces present are the weight force $F_g = m_p g$, the normal force F_N exerted by the ground on the feet, and the friction force F_h exerted horizontally by the ground on the feet. Here z is the height of the rope (assumed approximately equal to the height of the centre of mass of the person) and x is the horizontal separation between the centre of mass and the feet of the person; the feet themselves are assumed to be close together.

Taking torque about the feet gives $\sum \tau = zF_{G1} - xm_p g = 0$, yielding

$$F_{G1} = \frac{x}{z}m_p g = m_p g\tan\phi. \quad (4)$$

Here ϕ is the angle a line from the feet to the centre of mass of the person makes with the vertical. If a person of mass 80 kg has a moderate lean ϕ of 30°, then equation (4) gives F_{G1} as 0.454 kN. The number of people required to pull the raft off the rock $n \equiv F_G/F_{G1} \approx 2.38\ \text{kN}/0.454\ \text{kN} \approx 5.24$, which we round up to 6 persons. Although a mechanical advantage system is being used, a substantial number of people are still required.

The above calculation can be repeated to find n for the canoe and the kayak held against a rock by moving water, and results are given in table 1. After rounding n up to the next integer, $n = 1$ is obtained in both cases. If the water velocity is twice as fast ($2 \times 1.5\ \text{m s}^{-1}$), the required values of F_L, F_G and n would be increased by a factor of four for each boat.

A worksheet has been set up in an ancillary file (available from the electronic version of the journal) to calculate forces and tensions for situations associated with rafts, canoes and kayaks created by the interested reader.

Acknowledgments

We thank B Harris, T J Hittle and A Laakmann for help with photographs and R J Stephenson for commenting on this manuscript.

Received 11 April 2005, in final form 18 August 2005
doi:10.1088/0031-9120/41/2/004

References

[1] River Rescue 3 International, PO Box 519, Elk Grove, CA 95759-0519, USA, www.rescue3.com/nfpa.html

[2] American Canoe Association, 7432 Alban Station Boulevard, Suite B-226, Springfield, VA 22150-2311, USA, www.acanet.org/sei-swiftwater-rescue.htm

[3] A wealth of information on canoeing, kayaking and rafting in Great Britain can be found at the British Canoe Union website at www.bcu.org.uk/

[4] See for example:
Ferrero F (ed) 2003 *English Whitewater* (Bangor, Wales: Pesda Press)
Bandtock P 1996 *Whitewater Europe, Book One: North Alps* (Surbiton, UK: Rivers Publishing)
Williams T 2004 *Whitewater Classics: Fifty North American Rivers Picked by the Continent's Leading Paddlers* (Flagstaff, AZ: Funhong Press)

[5] A selection of companies that provide guided whitewater raft trips are:
www.liquidlife.co.uk/scotraft.html (England and Scotland)
www.activities-scotland.com/ (Fort William, Scotland)
www.aboard-rafting.com/ (Verdon, France)
www.jolster-rafting.no/informasjon/ordliste.php (Norway)

[6] Alpine Safety & Information Center (ASI-Tirol) Lantech, Bruggfeldstr. 5, A-6500 Landeck, Austria, www.alpinesicherheit.com

[7] Bechdel L and Ray S 1985 *River Rescue* (Boston, MA: Appalachian Mountain Club)

[8] Babinsky H 2003 How do wings work? *Phys. Educ.* **38** 497

[9] Binder R C 1973 *Fluid Mechanics* (Englewood Cliffs, NJ: Prentice Hall)

[10] Raleigh D 1998 *Knots and Ropes for Climbers* (Pennsylvania: Stackpole Books)

Michael J O'Shea received a PhD in physics from the University of Sussex and is currently a professor of physics at Kansas State University (USA), where he studies nanostructured rare-earth based materials. He is certified by the American Canoe Association to teach whitewater canoeing and works for Outward Bound Wilderness as an instructor and course director.